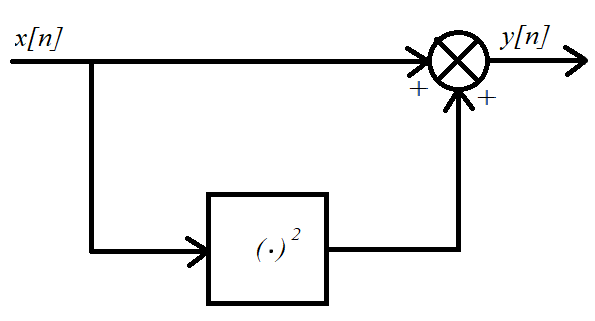
Bispectral Analysis

The bispectrum is a higher-order statistics tool that can be used to detect quadratic nonlinearities in a speech production system by detecting traces of quadratic phase coupling (QPC).

A quadratically nonlinear system, such as will introduce frequency components to the output which are not present in the input.



If the input to the system is

then the output is

This indicates that a quadratically nonlinear system may introduce frequency components at double the input frequencies and at sums/differences of the input frequencies.

|  |  |  |  |
| --- | --- | --- | --- |
| **Input frequencies** | **Input phases** | **Output frequencies** | **Output phases** |
|  |  | (DC component) |  |

The bispectrum is

where is the DTFT of and denotes the complex conjugate. The doubling of the input frequencies can be considered as a special case, when . The differencing of the input frequencies can be considered by taking the negative frequency axis (note that this only occurs with real signals which have a symmetrical spectrum).

# Bispectrum and Polyspectra

The power spectrum of is

which can be used to detect second order correlations, but is blind to higher order correlations. The bispectrum of is defined as

which can be used to detect third order correlations (quadratic nonlinearity). In general, the -th order polyspectrum is

which can be used to detect -th order correlations.

# Magnitude Response

For a ‘large’ magnitude, we require , and to all be large. If a signal happens to have significant frequency components at and is the output of a quadratically nonlinear system, then it is expected that a significant component will also be introduced at , causing a peak in the bispectrum at .

# Phase Response

The phase of ) is

Due to QPC, peaks in the bispectrum introduced due to quadratic nonlinearity will also have zero phase.

# Arbitrarily Nonlinear Systems

For an arbitrary nonlinear system, the system function can be decomposed via a Taylor series,

Disregarding the higher order terms,

This is the form of a quadratically nonlinear system, thus we expect some QPC (and also higher order correlations) in an arbitrary nonlinear speech production system.

# Bicoherence

The bicoherence is the normalised bispectrum

When calculating the bicoherence for a long speech signal, it can be approximated by chunking and windowing the signal, calculating the bicoherence of each windowed segment and averaging.

# MATLAB Implementation

clear;

addpath('..\Speech samples');

% Specify constants

N = 1024; % No. samples

M = N/2;

n = -M:M-1;

f = n/N;

B\_bins = 128; % No. bins for bispectrum plot

B\_bin\_size = N/B\_bins;

% Quadratically nonlinear system

f1 = 0.1;

f2 = 0.15;

x = exp(2j\*pi\*f1\*n)+exp(2j\*pi\*f2\*n);

y = x.^2 + x;

% Audio read

x = zeros(1,N);

y = 5\*audioread('synthetic\_vowel.wav');

%y = 5\*6\*audioread('vowel.wav');

y = y(1:N);

x = zeros(N,1);

% FFTs

X = 1/sqrt(N)\*fft(x);

Y = 1/sqrt(N)\*fft(y);

% Bispectrum calculation

for k=1:N

for m=1:N

if k+m<=N

B(k,m) = Y(k)\*Y(m)\*conj(Y(k+m));

else

B(k,m) = Y(k)\*Y(m)\*conj(Y(k+m-N));

end

end

end

% Make spectrum symmetrical around zero

X = fftshift(X);

Y = fftshift(Y);

B = fftshift(B);

% Bin the bispectrum, so that it is easier to see

B\_small = zeros(B\_bins);

B\_small = reshape(B,B\_bin\_size,B\_bins,B\_bin\_size,B\_bins,1);

B\_small = sum(B\_small,[1 3]);

B\_small = reshape(B\_small,B\_bins,B\_bins,1);

ticks = linspace(1.5, B\_bins-0.5, 11);

ticklabels = {'-0.5' '-0.4' '-0.3' '-0.2' '-0.1' '0' '0.1' '0.2' '0.3' '0.4' '0.5'};

% Plot input and output spectrum

figure(1)

subplot(2,1,1)

plot(f,abs(X))

title('Input magnitude response')

xlabel('f (normalised)')

ylabel('X[f]')

subplot(2,1,2)

plot(f,abs(Y))

title('Output magnitude response')

xlabel('f (normalised)')

ylabel('Y[f]')

% Surf plot of bispectrum magnitude

figure(2)

surf(abs(B\_small))

title('Bispectrum magnitude')

xlabel('f1 (normalised)')

ylabel('f2 (normalised)')

zlabel('|B[f1,f2]|')

xticks(ticks)

xticklabels(ticklabels)

yticks(ticks)

yticklabels(ticklabels)

% Contour plot of bispectrum

figure(3)

contour(abs(B\_small))

title('Bispectrum magnitude')

xlabel('f1 (normalised)')

ylabel('f2 (normalised)')

zlabel('|B[f1,f2]|')

xticks(ticks)

xticklabels(ticklabels)

yticks(ticks)

yticklabels(ticklabels)

% Surf plot of bispectrum phase

figure(4)

contour(angle(B\_small))

title('Bispectrum phase')

xlabel('f1 (normalised)')

ylabel('f2 (normalised)')

zlabel('|B[f1,f2]|')

xticks(ticks)

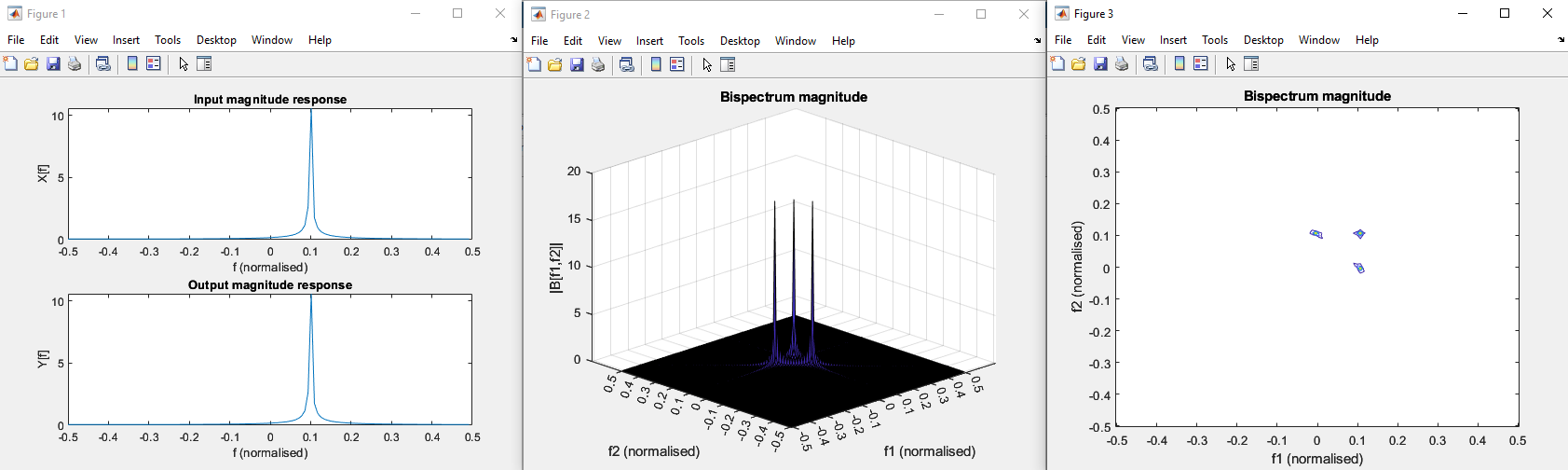
xticklabels(ticklabels)

yticks(ticks)

yticklabels(ticklabels)

# Single Complex Exponential

Here, the bispectrum is calculated for a single complex exponential where the normalised frequency is set to . The peaks in the bispectrum magnitude are around 18.

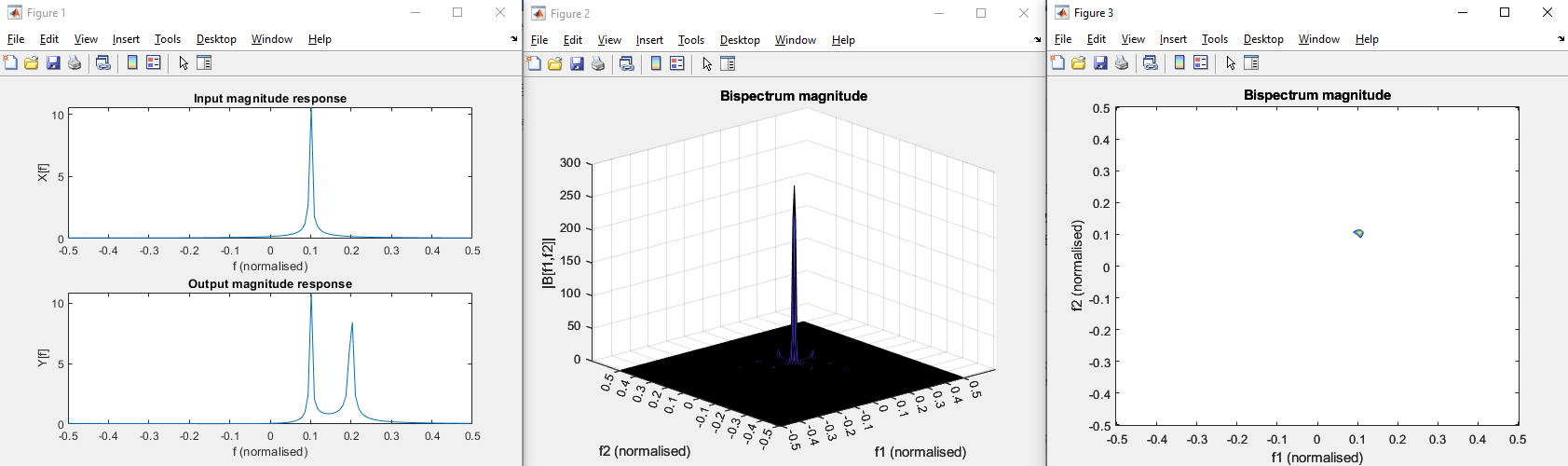


# Quadratically Nonlinear System with Single Complex Exponential Input

Now, feed the complex exponential through the system . The output is

|  |  |
| --- | --- |
|  | (8.10) |

so there are frequency components at and . Nonlinearity should be detected at Now, calculate the bispectrum of .



There is significant bispectrum magnitude at indicating high correlation between and as expected. The biphase appears to be zero at the peak – check this.

Updated Notes

The bispectrum is a tool used in higher-order statistics for detecting nonlinearities in systems, defined as the two dimensional Fourier transform of the third order cumulant,

where the third order cumulant is defined as